

Exercises for 2.1–2.3

1. Which of the following are wff?

- (a) P
- (b) $W \vee S$
- (c) $\{[Y \wedge (\wedge T)] \wedge L\} \wedge L$
- (d) $(P \wedge M) \vee \neg[(W \wedge) \vee (R \vee Q)]$
- (e) $Z \vee Z$
- (f) $Z \wedge [(S \vee R) \wedge P \vee L]$
- (g) $Z \wedge [(S \vee R) \wedge (P \vee L)]$
- (h) $[(Z \vee L) \wedge \neg Z] \vee (Q \wedge S)$
- (i) $L \vee Q$
- (j) $L \vee Q \vee R$
- (k) WR
- (l) $S \vee (S \wedge Y)$
- (m) $(L \wedge T) \wedge \{[T \vee (\{ \vee W \})] \vee (Q \vee M)\}$
- (n) $[(L \wedge M) \vee P] \wedge (Y \vee P$
- (o) $[(L \wedge P) \vee T] \vee \vee (M \vee L)$
- (p) $L \wedge \{(W \wedge W) \wedge [(W \wedge Z) \vee (Z \vee P)]\}$

2. Circle, highlight, or otherwise clearly mark the main connective of each of the following sentences.

- (a) $\{[(Z \wedge R) \wedge T] \wedge [(Y \wedge Y) \vee (S \wedge Z)]\} \vee [S \vee (T \vee Y)]$
- (b) $T \vee \neg W$
- (c) $\{[(W \wedge R) \wedge (L \wedge T)] \vee (P \wedge Z)\} \vee (Y \vee S)$
- (d) $[(Q \wedge Q) \wedge (R \vee Q)] \wedge [(L \wedge W) \vee T]$
- (e) $\neg R \vee \{(T \vee S) \wedge [\neg W \wedge (W \vee P)]\}$
- (f) $\neg M \wedge M$
- (g) $\neg(M \wedge M)$
- (h) $(L \wedge M) \vee \{[(T \vee S) \vee (Z \wedge T)] \wedge (W \wedge Y)\}$

- (i) $M \wedge (Q \wedge Z)$
- (j) $Q \vee S$
- (k) $\{[(Q \vee P) \wedge (Y \wedge Y)] \vee [(M \vee S) \wedge T]\} \vee Q$
- (l) $\neg Q$
- (m) $\left(\{[T \wedge (R \vee \neg Z)] \wedge Y\} \wedge (Z \wedge \neg T) \right) \wedge (S \vee L)$
- (n) $\neg P \wedge \{[(L \vee Y) \wedge (Z \wedge Q)] \vee [P \vee (M \wedge M)]\}$
- (o) $(Q \wedge W) \vee (T \wedge P)$
- (p) $(Q \wedge Q) \vee W$
- (q) $[S \vee (W \wedge Y)] \wedge \{[Y \vee (Z \wedge Q)] \wedge Q\}$
- (r) $\{(R \vee L) \wedge [(R \vee P) \wedge Y]\} \vee (Z \wedge Y)$
- (s) $[(Z \vee M) \vee (S \wedge R)] \wedge \{[(S \wedge W) \wedge (L \vee Q)] \wedge Q\}$

3. Which of the sentences (a) through (h) in the previous question are of the form

- (a) $s_1 \vee s_2$
- (b) $s_2 \wedge s_2$
- (c) $\neg s$

4. A well-formed formula (wff) is a string of symbols (logical connectives, opening and closing brackets, letters of the Roman alphabet possibly with some subscripts). Let's think a bit about how a wff must look like. For each of the following claim, state whether it's true or false of wffs:

- (a) A closing bracket cannot be the first symbol.
- (b) A closing bracket cannot be the last symbol.
- (c) A logical connective cannot be the first symbol.
- (d) The negation can be the first symbol.
- (e) \neg cannot be followed immediately by \wedge .
- (f) \wedge cannot be followed immediately by \neg .

5. Take the truth table for conjunction. Plug $(A \wedge B)$ into s_1 and $(A \vee B)$ into s_2 and produce the resulting truth conditions for $(A \wedge B) \wedge (A \vee B)$.