

## Exercises for 2.1–2.3

1. Which of the following are wff?
  - $P$
  - $W \vee S$
  - $\{[Y \wedge (\wedge T)] \wedge L\} \wedge L$
  - $(P \wedge M) \vee \neg[(W \wedge) \vee (R \vee Q)]$
  - $Z \vee Z$
  - $Z \wedge [(S \vee R) \wedge P \vee L]$
  - $Z \wedge [(S \vee R) \wedge (P \vee L)]$
  - $[(Z \vee L) \wedge \neg Z] \vee (Q \wedge S)$
  - $L \vee Q$
  - $L \vee Q \vee R$
  - $WR$
  - $S \vee (S \wedge Y)$
  - $(L \wedge T) \wedge \{[T \vee (\{\vee W\})] \vee (Q \vee M)\}$
  - $[(L \wedge M) \vee P] \wedge (Y \vee P)$
  - $[(L \wedge P) \vee T] \vee \vee (M \vee L)$
  - $L \wedge \{(W \wedge W) \wedge [(W \wedge Z) \vee (Z \vee P)]\}$
2. Circle, highlight, or otherwise clearly mark the main connective of each of the following sentences.
  - $\{[(Z \wedge R) \wedge T] \wedge [(Y \wedge Y) \vee (S \wedge Z)]\} \vee [S \vee (T \vee Y)]$
  - $T \vee \neg W$
  - $\{[(W \wedge R) \wedge (L \wedge T)] \vee (P \wedge Z)\} \vee (Y \vee S)$
  - $[(Q \wedge Q) \wedge (R \vee Q)] \wedge [(L \wedge W) \vee T]$
  - $\neg R \vee \{(T \vee S) \wedge [\neg W \wedge (W \vee P)]\}$
  - $\neg M \wedge M$
  - $\neg(M \wedge M)$
  - $(L \wedge M) \vee \{[(T \vee S) \vee (Z \wedge T)] \wedge (W \wedge Y)\}$

(i)  $M \wedge (Q \wedge Z)$   
 (j)  $Q \vee S$   
 (k)  $\{(Q \vee P) \wedge (Y \wedge Y)\} \vee [(M \vee S) \wedge T] \vee Q$   
 (l)  $\neg Q$   
 (m)  $\left(\{[T \wedge (R \vee \neg Z)] \wedge Y\} \wedge (Z \wedge \neg T)\right) \wedge (S \vee L)$   
 (n)  $\neg P \wedge \{[(L \vee Y) \wedge (Z \wedge Q)] \vee [P \vee (M \wedge M)]\}$   
 (o)  $(Q \wedge W) \vee (T \wedge P)$   
 (p)  $(Q \wedge Q) \vee W$   
 (q)  $[S \vee (W \wedge Y)] \wedge \{[Y \vee (Z \wedge Q)] \wedge Q\}$   
 (r)  $\{(R \vee L) \wedge [(R \vee P) \wedge Y]\} \vee (Z \wedge Y)$   
 (s)  $[(Z \vee M) \vee (S \wedge R)] \wedge \{[(S \wedge W) \wedge (L \vee Q)] \wedge Q\}$

3. Which of the sentences (a) through (h) in the previous question are of the form

(a)  $s_1 \vee s_2$   
 (b)  $s_2 \wedge s_2$   
 (c)  $\neg s$

4. A well-formed formula (wff) is a string of symbols (logical connectives, opening and closing brackets, letters of the Roman alphabet possibly with some subscripts). Let's think a bit about how a wff must look like. For each of the following claim, state whether it's true or false of wffs:

(a) A closing bracket cannot be the first symbol.  
 (b) A closing bracket cannot be the last symbol.  
 (c) A logical connective cannot be the first symbol.  
 (d) The negation can be the first symbol.  
 (e)  $\neg$  cannot be followed immediately by  $\wedge$ .  
 (f)  $\wedge$  cannot be followed immediately by  $\neg$ .

5. Take the truth table for conjunction. Plug  $(A \wedge B)$  into  $s_1$  and  $(A \vee B)$  into  $s_2$  and produce the resulting truth conditions for  $(A \wedge B) \wedge (A \vee B)$ .