Exercises for 2.4–2.6

1. Produce the syntax tree for each of the following:

   (a) $(P \wedge Q) \wedge (P \vee Q)$
   (b) $(S \wedge T) \vee \neg R$
   (c) $P \wedge (Q \wedge R)$
   (d) $(R \vee S) \wedge T$
   (e) $R \vee (S \wedge T)$
   (f) $\neg(P \vee R) \vee (Q \wedge S)$

2. Give the truth table for $P \wedge (Q \wedge R)$.

3. Give the truth table for $(R \vee S) \wedge T$.

4. Give the truth table for $R \vee (S \wedge T)$.

5. Give the truth table for $\neg(P \wedge Q)$.

6. Give the truth table for $\neg P \vee \neg Q$.

7. Give the truth table for $\neg(R \vee S)$.

8. Give the truth table for $\neg R \wedge \neg S$.

9. Show, using a truth table, that $P \vee \neg P$ is a tautology.

10. Show, using a truth table, that $P \wedge \neg P$ is a contradiction.

11. Explain why any sentence of the form $s \vee \neg s$ is a tautology (even if s is not an atomic sentence).

12. True or False?

    (a) If one disjunct of a disjunction is a tautology, then the whole disjunction is a tautology.
    ..... True/False
    (b) If a disjunction is a tautology, one of the disjuncts is a tautology.
    ..... True/False
    (c) The negation of a contradiction is a tautology.
    ..... True/False

(d) $s_1 \wedge s_2$ is consistent iff. s_1 and s_2 are consistent with each other.

. . . . . True/False

(e) If s_1 and s_2 are inconsistent with each other, then $s_1 \vee s_2$ is inconsistent.

. . . . . True/False