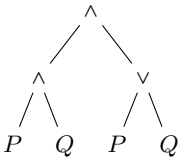
Exercises for 2.4–2.6

1. Produce the syntax tree for each of the following:

   (a) $(P \wedge Q) \wedge (P \vee Q)$

   Answer Key

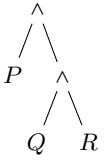
   (b) $(S \wedge T) \vee \neg R$

   Answer Key

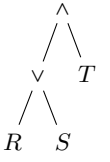
   (c) $P \wedge (Q \wedge R)$

   Answer Key

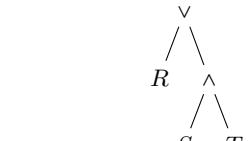
   

   (d) $(R \vee S) \wedge T$

   Answer Key

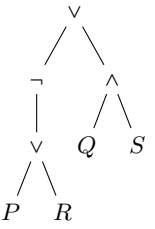
   

   (e) $R \vee (S \wedge T)$



Answer Key

(f) $\neg(P \vee R) \vee (Q \wedge S)$



Answer Key

2. Give the truth table for $P \wedge (Q \wedge R)$.

Answer Key

P	Q	R	$Q \wedge R$	$P \wedge (Q \wedge R)$
T	*T*	*T*	*T*	*T*
F	*T*	*T*	*T*	*F*
T	*F*	*T*	*F*	*F*
F	*F*	*T*	*F*	*F*
T	*T*	*F*	*F*	*F*
F	*T*	*F*	*F*	*F*
T	*F*	*F*	*F*	*F*
F	*F*	*F*	*F*	*F*

.

3. Give the truth table for $(R \vee S) \wedge T$.

Answer Key

R	S	T	$R \vee S$	$(R \vee S) \wedge T$
T	T	T	T	T
F	T	T	T	T
T	F	T	T	T
F	F	T	F	F
T	T	F	T	F
F	T	F	T	F
T	F	F	T	F
F	F	F	F	F

4. Give the truth table for $R \vee (S \wedge T)$.

Answer Key

R	S	T	$S \wedge T$	$R \vee (S \wedge T)$
T	T	T	T	T
F	T	T	T	T
T	F	T	F	T
F	F	T	F	F
T	T	F	F	T
F	T	F	F	F
T	F	F	F	T
F	F	F	F	F

5. Give the truth table for $\neg(P \wedge Q)$.

Answer Key

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	T	F
F	T	F	T
T	F	F	T
F	F	F	T

6. Give the truth table for $\neg P \vee \neg Q$.

Answer Key

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	F	F	F
F	T	T	F	T
T	F	F	T	T
F	F	T	T	T

7. Give the truth table for $\neg(R \vee S)$.

Answer Key

R	S	$R \vee S$	$\neg(R \vee S)$
T	T	T	F
F	T	T	F
T	F	T	F
F	F	F	T

8. Give the truth table for $\neg R \wedge \neg S$.

Answer Key

R	S	$\neg R$	$\neg S$	$\neg R \wedge \neg S$
T	T	F	F	F
F	T	T	F	F
T	F	F	T	F
F	F	T	T	T

9. Show, using a truth table, that $P \vee \neg P$ is a tautology.

Answer Key

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

10. Show, using a truth table, that $P \wedge \neg P$ is a contradiction.

Answer Key

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

11. Explain why any sentence of the form $s \vee \neg s$ is a tautology (even if s is not an atomic sentence).

Answer Key

Given any interpretation, s is either true of false. If s is true in that interpretation, $s \vee \neg s$ is also true in that interpretation. If s is false, then $\neg s$ is true, so $s \vee \neg s$ is true in that interpretation. So $s \vee \neg s$ is true in every interpretation, which means that $s \vee \neg s$ is a tautology.

12. True or False?

    (a) If one disjunct of a disjunction is a tautology, then the whole disjunction is a tautology.

    ..... (True)/False

(b) If a disjunction is a tautology, one of the disjuncts is a tautology.
. . . . . True/(False)

(c) The negation of a contradiction is a tautology.
. . . . . (True)/False

(d) $s_1 \wedge s_2$ is consistent iff. s_1 and s_2 are consistent with each other.
. . . . . (True)/False

(e) If s_1 and s_2 are inconsistent with each other, then $s_1 \vee s_2$ is inconsistent.
. . . . . True/(False)