Exercises for 2.7

1. I say in the text that $s_1 \supset s_2$ can be read as 's_1 only if s_2'. Explain why, given the truth table of $s_1 \supset s_2$, it makes sense to read is as 's_1 only if s_2'.

> **Answer Key**
>
> Suppose someone asserts $s_1 \supset s_2$. What they assert is that the world is such that ⟨both s_1 and s_2 are true (1st valuation), or s_1 is false but s_2 is true (2nd valuation), or both s_1 and s_2 are false (4th valuation)⟩. We can see that given the truth of the conditional, the falsity of s_2 guarantees the falsity of s_1. So the truth of s_2 is necessary for the truth of s_1 and that is what 's_1 only if s_2' means.

2. One way of reading $s_1 \supset s_2$ that I do not give in the text is 's_2 if s_1'. Explain why this is also a good way of reading $s_1 \supset s_2$.

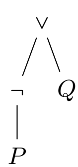
> **Answer Key**
>
> We have seen above that $s_1 \supset s_2$ says that the truth of s_2 is necessary for the truth of s_1. This means that given the truth of $s_1 \supset s_2$, it cannot be that the s_2 is false even though s_1 is true. Hence, the truth of s_1 is sufficient for the truth of s_2. And that is what 's_2 if s_1' says.

3. How could we say 's_1 if and only if s_2' by utilizing the availability of $\supset$?

> **Answer Key**
>
> $(s_1 \supset s_2) \wedge (s_2 \supset s_1)$

4. Give the syntax tree and truth table of each of the following sentences.

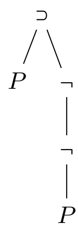
   (a) $\neg P \vee Q$

> **Answer Key**
>
> 
>

P	Q	$\neg P$	$\neg P \vee Q$
T	T	F	T
F	T	T	T
T	F	F	F
F	F	T	T

(b) $P \supset \neg\neg P$

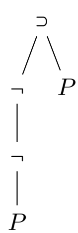
Answer Key



P	$\neg P$	$\neg\neg P$	$P \supset \neg\neg P$
T	F	T	T
F	T	F	T

(c) $\neg\neg P \supset P$

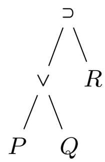
Answer Key



P	$\neg P$	$\neg\neg P$	$\neg\neg P \supset P$
T	F	T	T
F	T	F	T

(d) $(P \vee Q) \supset R$

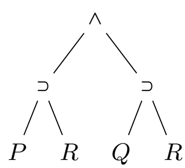
Answer Key



P	Q	R	$P \vee Q$	$(P \vee Q) \supset R$
T	T	T	T	T
F	T	T	T	T
T	F	T	T	T
F	F	T	F	T
T	T	F	T	F
F	T	F	T	F
T	F	F	T	F
F	F	F	F	T

(e) $(P \supset R) \wedge (Q \supset R)$

Answer Key



P	Q	R	$P \supset R$	$Q \supset R$	$(P \supset R) \wedge (Q \supset R)$
T	T	T	T	T	T
F	T	T	T	T	T
T	F	T	T	T	T
F	F	T	T	T	T
T	T	F	F	F	F
F	T	F	T	F	F
T	F	F	F	T	F
F	F	F	T	T	T

5. Show, using truth tables, that $s_1 \supset s_2$ is logically equivalent to $\neg s_1 \vee s_2$.

Answer Key

Here is the truth table for $\neg s_1 \vee s_2$:

s_1	s_2	$\neg s_1$	$\neg s_1 \vee s_2$
T	T	F	T
F	T	T	T
T	F	F	F
F	F	T	T

6. Explain why the following is true: If s_1 and s_2 are logically equivalent, then $s_1 \supset s_2$ is a tautology.

Answer Key

The truth values of s_1 and s_2 are the same in every interpretation. Given the truth table of the conditional, if both s_1 and s_2 are true, $s_1 \supset s_2$ is true. If both are false, $s_1 \supset s_2$ is also true. So the conditional is true in every interpretation which means it is a tautology.