1. Fill in missing items:

1.	Γ	$\vdash (P \vee Q) \supset R$	premise
2.	P	$\vdash P$	A
3.	__	$\vdash$ __	2,∨I
4.	__	$\vdash$ __	1,3,⊃E
5.	Γ	$\vdash$ __	4,⊃I
6.	Q	$\vdash Q$	A
7.	__	$\vdash$ __	6,∨I
8.	__	$\vdash$ __	1,7,⊃E
9.	Γ	$\vdash Q \supset R$	8,⊃I
10.	Γ	$\vdash$ __	5,9,∧I

2. Add missing items.

1.	Γ	$\vdash \neg P \wedge \neg Q$	premise
2.	$P \vee Q$	$\vdash P \vee Q$	A
3.	P	$\vdash$ __	A
4.	Γ	$\vdash \neg P$	__
5.	$P, P \vee Q$	$\vdash P$	__
6.	Γ, __	$\vdash \neg P$	4
7.	__	$\vdash \neg(P \vee Q)$	5,6,¬I
8.	__	$\vdash Q$	A
9.	Γ	$\vdash \neg Q$	__
10.	Q, __	$\vdash Q$	8
11.	Γ, __	$\vdash \neg Q$	9
12.	__	$\vdash \neg(P \vee Q)$	10,11,¬I
13.	$\Gamma, P \vee Q$	$\vdash \neg(P \vee Q)$	__
14.	Γ	$\vdash \neg(P \vee Q)$	__

3. Here is part of a derivation from $\Gamma \vdash \neg(P \vee Q)$ to $\Gamma \vdash \neg P \wedge \neg Q$. Complete the rest.

1.	Γ	$\vdash \neg(P \vee Q)$	.... premise
2.	P	$\vdash P$	.... A
3.	P	$\vdash P \vee Q$	.... 2,∨I
4.	Γ, P	$\vdash \neg(P \vee Q)$	 1
5.	Γ	$\vdash \neg P$	.... 3,4,¬I

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4. When someone offers considerations that lead to a contradiction, that is usually taken to be a bad thing. One reason why contradictions are bad is captured by the observation known as *ex contradictione quodlibet*: from a contradiction, derive at will. That is, if you had proof of a contradiction you could prove anything you want. The following demonstrates the point. Add the missing annotations:

1.	Γ	$\vdash P \wedge \neg P$	.... ___
2.	$\Gamma, \neg Q$	$\vdash P \wedge \neg P$	.... ___
3.	$\Gamma, \neg Q$	$\vdash P$	.... ___
4.	$\Gamma, \neg Q$	$\vdash \neg P$	.... ___
5.	Γ	$\vdash \neg\neg Q$	.... ___
6.	Γ	$\vdash Q$	.... ___

Notice that you could replace Q with anything you please. So can equally well derive $\neg Q$. Here we have a decisive reason to reject the premise: something must have gone wrong in thinking that we have conclusive reason to accept the premise.

5. Derive from $\Gamma \vdash P \lor P$ to $\Gamma \vdash P$.

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6. Derive from $\Gamma \vdash P \supset (Q \supset R)$ to $\Gamma \vdash (P \land Q) \supset R$. Hint: assume $P \land Q$.

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7. Derive from $\Gamma \vdash (P \land Q) \supset R$ to $\Gamma \vdash P \supset (Q \supset R)$. Hint: assume P and assume Q.

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8. We noted earlier that the conditional ($\supset$) has some odd features. The oddities show up in our proof system as well.

   (a) Derive from $\Gamma \vdash P$ to $\Gamma \vdash Q \supset P$. (Hint: remember you can add anything you want to the datum of a sequent).

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   (b) Derive from $\Gamma \vdash \neg P$ to $\Gamma \vdash P \supset Q$. (Hint: assume P, and remember you can add anything you want, in particular $\neg Q$ to the datum—see also the problem at the top of these exercises.)

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   (c) Derive from $\Gamma \vdash P$ to $\Gamma \vdash \neg P \supset Q$. (Hint: assume $\neg P$; and don't forget the point about being able to add things to the datum.)

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9. Derive from $\Gamma \vdash P \supset (Q \vee R)$ and $\Delta \vdash \neg Q$ to $\Gamma, \Delta \vdash P \supset R$. (Hint: First derive $\Gamma, P \vdash Q \vee R$. Then adapt the derivation in the first problem of the previous set of exercises.)