1. Fill in missing items:

1.	Γ	$\vdash$	$(P \vee Q) \supset R$	premise
2.	P	$\vdash$	P	A
3.	$\underline{P}$	$\vdash$	$\underline{P \vee Q}$	2,$\vee$I
4.	$\underline{\Gamma, P}$	$\vdash$	$\underline{R}$	1,3,$\supset$E
5.	Γ	$\vdash$	$\underline{P \supset R}$	4,$\supset$I
6.	Q	$\vdash$	Q	A
7.	$\underline{Q}$	$\vdash$	$\underline{P \vee Q}$	6,$\vee$I
8.	$\underline{\Gamma, Q}$	$\vdash$	$\underline{R}$	1,7,$\supset$E
9.	Γ	$\vdash$	$Q \supset R$	8,$\supset$I
10.	Γ	$\vdash$	$\underline{(P \supset R) \wedge (Q \supset R)}$	5,9,$\wedge$I

2. Add missing items.

1.	Γ	$\vdash$	$\neg P \wedge \neg Q$	premise
2.	$P \vee Q$	$\vdash$	$P \vee Q$	A
3.	P	$\vdash$	$\underline{P}$	A
4.	Γ	$\vdash$	$\neg P$	$\underline{1,\wedge\text{E}}$
5.	$P, P \vee Q$	$\vdash$	P	$\underline{3}$
6.	$\Gamma, \underline{P \vee Q}$	$\vdash$	$\neg P$	4
7.	$\underline{\Gamma, P}$	$\vdash$	$\neg(P \vee Q)$	5,6,$\neg$I
8.	$\underline{Q}$	$\vdash$	Q	A
9.	Γ	$\vdash$	$\neg Q$	$\underline{1,\wedge\text{E}}$
10.	$Q, \underline{P \vee Q}$	$\vdash$	Q	8
11.	$\Gamma, \underline{P \vee Q}$	$\vdash$	$\neg Q$	9
12.	$\underline{\Gamma, Q}$	$\vdash$	$\neg(P \vee Q)$	10,11,$\neg$I
13.	$\Gamma, P \vee Q$	$\vdash$	$\neg(P \vee Q)$	$\underline{2,7,12,\vee\text{E}}$
14.	Γ	$\vdash$	$\neg(P \vee Q)$	$\underline{2,13,\neg\text{I}}$

3. Here is part of a derivation from $\Gamma \vdash \neg(P \vee Q)$ to $\Gamma \vdash \neg P \wedge \neg Q$. Complete the rest.

1.	Γ	$\vdash \neg(P \vee Q)$	premise
2.	P	$\vdash P$	A
3.	P	$\vdash P \vee Q$	2,∨I
4.	Γ, P	$\vdash \neg(P \vee Q)$	1
5.	Γ	$\vdash \neg P$	3,4,¬I

Answer Key

6.	Q	$\vdash Q$	A
7.	Q	$\vdash P \vee Q$	6,∨I
8.	Γ, Q	$\vdash \neg(P \vee Q)$	1
9.	Γ	$\vdash \neg Q$	7,8,¬I
10.	Γ	$\vdash \neg P \wedge \neg Q$	5,9,∧I

4. When someone offers considerations that lead to a contradiction, that is usually taken to be a bad thing. One reason why contradictions are bad is captured by the observation known as *ex contradictione quodlibet*: from a contradiction, derive at will. That is, if you had proof of a contradiction you could prove anything you want. The following demonstrates the point. Add the missing annotations:

1.	Γ	$\vdash P \wedge \neg P$	premise
2.	$\Gamma, \neg Q$	$\vdash P \wedge \neg P$	1
3.	$\Gamma, \neg Q$	$\vdash P$	2,∧E
4.	$\Gamma, \neg Q$	$\vdash \neg P$	2,∧E
5.	Γ	$\vdash \neg\neg Q$	3,4,¬I
6.	Γ	$\vdash Q$	5,¬E

Notice that you could replace Q with anything you please. So can equally well derive $\neg Q$. Here we have a decisive reason to reject the premise: something must have gone wrong in thinking that we have conclusive reason to accept the premise.

5. Derive from $\Gamma \vdash P \vee P$ to $\Gamma \vdash P$.

Answer Key

1.	Γ	$\vdash P \vee P$	premise
2.	P	$\vdash P$	A
3.	Γ	$\vdash P$	1,2,2,$\vee$E

6. Derive from $\Gamma \vdash P \supset (Q \supset R)$ to $\Gamma \vdash (P \wedge Q) \supset R$. Hint: assume $P \wedge Q$.

Answer Key

1.	Γ	$\vdash P \supset (Q \supset R)$	premise
2.	$P \wedge Q$	$\vdash P \wedge Q$	A
3.	$P \wedge Q$	$\vdash P$	2,$\wedge$E
4.	$\Gamma, P \wedge Q$	$\vdash Q \supset R$	1,3,$\supset$E
5.	$P \wedge Q$	$\vdash Q$	2,$\wedge$E
6.	$\Gamma, P \wedge Q$	$\vdash R$	4,5,$\supset$E
7.	Γ	$\vdash (P \wedge Q) \supset R$	6,$\supset$I

7. Derive from $\Gamma \vdash (P \wedge Q) \supset R$ to $\Gamma \vdash P \supset (Q \supset R)$. Hint: assume P and assume Q.

Answer Key

1. $\Gamma \vdash (P \wedge Q) \supset R$ .... premise
2. $P \vdash P$ .... A
3. $Q \vdash Q$ .... A
4. $P, Q \vdash P \wedge Q$ .... 2,3,∧I
5. $\Gamma, P, Q \vdash R$ .... 1,4,⊃E
6. $\Gamma, P \vdash Q \supset R$ .... 5,⊃I
7. $\Gamma \vdash P \supset (Q \supset R)$ .... 6,⊃I

8. We noted earlier that the conditional (⊃) has some odd features. The oddities show up in our proof system as well.

   (a) Derive from $\Gamma \vdash P$ to $\Gamma \vdash Q \supset P$. (Hint: remember you can add anything you want to the datum of a sequent).

   Answer Key

   1. $\Gamma \vdash P$ .... premise
   2. $\Gamma, Q \vdash P$ 1
   3. $\Gamma \vdash Q \supset P$ .... 2,⊃I

   (b) Derive from $\Gamma \vdash \neg P$ to $\Gamma \vdash P \supset Q$. (Hint: assume P, and remember you can add anything you want, in particular $\neg Q$ to the datum—see also the problem at the top of these exercises.)

   Answer Key

   1. $\Gamma \vdash \neg P$ .... premise
   2. $P \vdash P$ .... A
   3. $P, \neg Q \vdash P$ 2
   4. $\Gamma, \neg Q \vdash \neg P$ 1
   5. $\Gamma, P \vdash \neg\neg Q$ .... 3,4,¬I
   6. $\Gamma, P \vdash Q$ .... 5,¬E
   7. $\Gamma \vdash P \supset Q$ .... 6,⊃I

(c) Derive from $\Gamma \vdash P$ to $\Gamma \vdash \neg P \supset Q$. (Hint: assume $\neg P$; and don't forget the point about being able to add things to the datum.)

Answer Key

1.	Γ	$\vdash P$	premise
2.	$\neg P$	$\vdash \neg P$	A
3.	$\Gamma, \neg Q$	$\vdash P$	1
4.	$\neg P, \neg Q$	$\vdash \neg P$	2
5.	$\Gamma, \neg P$	$\vdash \neg\neg Q$	3,4,¬I
6.	$\Gamma, \neg P$	$\vdash Q$	5,¬E
7.	Γ	$\vdash \neg P \supset Q$	6,⊃I

9. Derive from $\Gamma \vdash P \supset (Q \vee R)$ and $\Delta \vdash \neg Q$ to $\Gamma, \Delta \vdash P \supset R$. (Hint: First derive $\Gamma, P \vdash Q \vee R$. Then adapt the derivation in the first problem of the previous set of exercises.)

Answer Key

1.	Γ	$\vdash P \supset (Q \vee R)$	premise
2.	Δ	$\vdash \neg Q$	premise
3.	P	$\vdash P$	A
4.	Γ, P	$\vdash Q \vee R$	1,3,⊃E
5.	Q	$\vdash Q$	A
6.	$\Delta, \neg R$	$\vdash \neg Q$	2
7.	$Q, \neg R$	$\vdash Q$	5
8.	Δ, Q	$\vdash \neg\neg R$	6,7,¬I
9.	Δ, Q	$\vdash R$	8,¬E
10.	R	$\vdash R$	A
11.	Γ, Δ, P	$\vdash R$	4,9,10,∨E
12.	Γ, Δ	$\vdash P \supset R$	11,⊃I