Exercises for 4.1–4.4

1. Fill in missing items:

1.	Γ	$\vdash (P \vee Q) \supset R$	..... premise
2.	P	$\vdash P$	..... A
3.	__	$\vdash$ __	..... 2,$\vee$I
4.	__	$\vdash$ __	..... 1,3,$\supset$E
5.	Γ	$\vdash$ __	..... 4,$\supset$I
6.	Q	$\vdash Q$	..... A
7.	__	$\vdash$ __	..... 6,$\vee$I
8.	__	$\vdash$ __	..... 1,7,$\supset$E
9.	Γ	$\vdash Q \supset R$	..... 8,$\supset$I
10.	Γ	$\vdash$ __	..... 5,9,$\wedge$I

2. Add missing annotations for the following proof of $\vdash (\neg P \vee \neg Q) \supset \neg(P \wedge Q)$

1.	$\neg P \vee \neg Q$	$\vdash \neg P \vee \neg Q$	..... A
2.	$P \wedge Q$	$\vdash P \wedge Q$	..... __
3.	$\neg P$	$\vdash \neg P$	..... __
4.	$P \wedge Q$	$\vdash P$	..... __
5.	$\neg P, P \wedge Q$	$\vdash \neg P$	..... __
6.	$\neg P$	$\vdash \neg(P \wedge Q)$	..... __
7.	$\neg Q$	$\vdash \neg Q$	..... __
8.	$P \wedge Q$	$\vdash Q$	..... __
9.	$\neg Q, P \wedge Q$	$\vdash \neg Q$	..... __
10.	$\neg Q$	$\vdash \neg(P \wedge Q)$	..... __
11.	$\neg P \vee \neg Q$	$\vdash \neg(P \wedge Q)$	..... __
12.		$\vdash (\neg P \vee \neg Q) \supset \neg(P \wedge Q)$	..... __

3. Prove $\vdash \neg(P \supset Q) \supset \neg Q$. (Hint: assume $\neg(P \supset Q)$ as well as Q .)

4. Here is part of a proof of $\neg(P \vee Q) \vdash \neg P \wedge \neg Q$. Complete the rest.

1. $\neg(P \vee Q)$	$\vdash \neg(P \vee Q)$	A
2. P	$\vdash P$	A
3. P	$\vdash P \vee Q$	2,$\vee$I
4. $\neg(P \vee Q), P$	$\vdash \neg(P \vee Q)$	1
5. $\neg(P \vee Q)$	$\vdash \neg P$	3,4,$\neg$I

5. Add missing items.

1.	$\neg(P \supset Q)$	$\vdash \neg(P \supset Q)$	A
2.	$\neg P$	$\vdash \neg P$	A
3.	__	$\vdash$ __	2
4.	__	$\vdash$ __	A
5.	__	$\vdash$ __	4
6.	$\neg P, P$	$\vdash \neg\neg Q$	3,5,¬I
7.	$\neg P, P$	$\vdash Q$	6,¬E
8.	__	$\vdash$ __	__
9.	$\neg(P \supset Q)$, __	$\vdash \neg(P \supset Q)$	1
10.	$\neg(P \supset Q)$	$\vdash \neg\neg P$	8,9,¬I
11.	$\neg(P \supset Q)$	$\vdash P$	__
12.		$\vdash \neg(P \supset Q) \supset P$	__

6. Prove $P \supset Q, P \vee Q \vdash Q$. Hint: Assume $P \supset Q$ and assume Q. Use $\vee$E.

7. Recall the Prisoner's Dilemma (exercise for 3.1-4). Let P mean that Jerry will confess, and let Q mean that Ben is better off confessing. Turn the reasoning in the standardized form provided in the answer key into a derivation that utilizes EM (hint: the derivation will have two premises).