Exercises for 4.5

1. Suppose there is a derivation from $\Gamma \vdash s$ to $\Gamma \vdash t$. Explain why it follows that $\vdash s \supset t$. (Hint: you can plug anything you want into Γ.)

   Answer Key

   We can plug s into Γ. The supposition gives us that we can prove $s \vdash t$. We can then use $\supset$I to get $\vdash s \supset t$.

2. Suppose $s \supset t$ is a tautology. Explain why in that case the truth of s guarantees the truth of t.

   Answer Key

   If the truth of s did not guarantee the truth of t, $s \supset t$ could be false and therefore would not be a tautology which contradicts the supposition.

3. Suppose we can show that if $\vdash s \supset t$, then $s \supset t$ is a tautology. Explain why this would show that if there is a derivation from $\Gamma \vdash s$ to $\Gamma \vdash t$, then the truth of s guarantees the truth of t.

   Answer Key

   From Q1 we know that if there is a derivation from $\Gamma \vdash s$ to $\Gamma \vdash t$, then $\vdash s \supset t$. Given our supposition for this question, this means that $s \supset t$ is a tautology. But then by Q2, it follows that the truth of s guarantees the truth of t.

4. Suppose we can show that if $s \supset t$ is a tautology, then $\vdash s \supset t$. Explain why this would show that if $s \supset t$ is a tautology, then there is a derivation from $\Gamma \vdash s$ to $\Gamma \vdash t$.

   Answer Key

   We can construct the following derivation:

1.	Γ	$\vdash s$	premise
2.		$\vdash s \supset t$	theorem
3.	Γ	$\vdash t$	1,2,$\supset$E

   By the supposition, if $s \supset t$ is a tautology, line 2 in the above derivation is

correct.