1. Show that Disjunction Elimination is a valid rule of inference.

Answer Key

Here is one way.

We are trying to show if $\Lambda_1 \vDash s_1 \vee s_2$, $\Lambda_2, s_1 \vDash s_3$, and $\Lambda_3, s_2 \vDash s_3$, then $\Lambda_1, \Lambda_2, \Lambda_3 \vDash s_3$.

Given $\Lambda_2, s_1 \vDash s_3$, we get $\Lambda_2 \vDash s_1 \supset s_3$. And given $\Lambda_3, s_2 \vDash s_3$, we get $\Lambda_3 \vDash s_2 \supset s_3$. But this means that the truth of all of $\Lambda_1, \Lambda_2, \Lambda_3$ guarantees the truth of s_3.

So $\Lambda_1, \Lambda_2, \Lambda_3 \vDash s_3$.

2. Show that Negation Introduction is a valid rule of inference.

Answer Key

We are trying to show that if $\Lambda_1, s_1 \vdash s_2$ and $\Lambda_2, s_1 \vdash \neg s_2$ are both correct, then so is $\Lambda_1, \Lambda_2 \vdash \neg s_1$.

Consider the following derivation:

1.	Λ_1, s_1	$\vdash s_2$	
2.	Λ_2, s_1	$\vdash \neg s_2$	
3.	Λ_1	$\vdash s_1 \supset s_2$	1,⊃I
4.	Λ_2	$\vdash s_1 \supset s_2$	2,⊃I
5.	Λ_1, Λ_2	$\vdash (s_1 \supset s_2) \wedge (s_1 \supset \neg s_2)$	3,4, ∧I

Since ⊃I and ∧I are both valid rules of inference, given the first two sequents are correct, we know that so is the last:

$\Lambda_1, \Lambda_2 \vDash (s_1 \supset s_2) \wedge (s_1 \supset \neg s_2)$

We can use truth tables to show: $\vDash [(s_1 \supset s_2) \wedge (s_1 \supset \neg s_2)] \supset \neg s_1$

Therefore,

$\Lambda_1, \Lambda_2 \vDash \neg s_1$

So ¬I is valid.