Exercises for 5.11-5.12

1. Add missing items (Hint: lines 6 and 11 are one of the more useful theorems of sentential logic):

1.	$\neg\forall x \neg Fx$	$\vdash \neg\forall x \neg Fx$	A
2.	$\neg\exists x Fx$	$\vdash \neg\exists x Fx$	A
3.	Fa	$\vdash Fa$	A
4.	Fa	$\vdash \exists x Fx$	__
5.		$\vdash$ __	4,⊃I
6.		$\vdash$ __	__
7.		$\vdash \neg\exists x Fx \supset \neg Fa$	5,6,⊃E
8.	$\neg\exists x Fx$	$\vdash \neg Fa$	__
9.	$\neg\exists x Fx$	$\vdash$ __	8,∀I
10.		$\vdash \neg\exists x Fx \supset \forall x \neg Fx$	__
11.		$\vdash$ __	__
12.		$\vdash \neg\forall x \neg Fx \supset \neg\neg\exists x Fx$	10,11, ⊃E
13.	$\neg\forall x \neg Fx$	$\vdash \neg\neg\exists x Fx$	__
14.	$\neg\forall x \neg Fx$	$\vdash \exists x Fx$	13,¬E

2. Suppose it is true that $\forall x[(Fx \vee Gx) \supset Hx]$, and suppose that Fa, where a is a constant. It follows that Ha. Turn this into a derivation. Here are the first two lines:

1.	Γ	$\vdash \forall x[(Fx \vee Gx) \supset Hx]$	premise
2.	Δ	$\vdash Fa$	premise

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3. Suppose everyone gets grumpy when hungry. So if everyone is hungry, everyone is grumpy. Turn this into a derivation. Here are the first two lines (Hx means x is hungry, Gx means x is grumpy; the conclusion you want to reach is $\Gamma \vdash \forall x Hx \supset \forall x Gx$):

    1. $\Gamma \quad \vdash \forall x(Hx \supset Gx)$ .... premise
    2. $\forall x Hx \quad \vdash \forall x Hx$ .... A

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4. Consider: No one is flawless; so everyone has flaws. Let's turn this into a derivation. Let Fx mean 'x has flaws' and let k be a constant. Hint: assume that k is not F.

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5. Consider: dragons are mythical creatures; but there are no mythical creatures; thus, there are no dragons. Formalize this. Let Dx mean that x is a dragon, Mx mean that x is a mythical creature. Hint: derive something that enables you to use ¬I.

1.	Γ	$\vdash \forall x(Dx \supset Mx)$	premise
2.	Δ	$\vdash \neg\exists x Mx$	premise
3.	$\exists x Dx$	$\vdash \exists x Dx$	A
4.	Da	$\vdash Da$	A
5.	Γ	$\vdash Da \supset Ma$	1,∀E

6. Prove $\vdash \forall x(Fx \supset \forall y Gy) \supset \forall x \forall y(Fx \supset Gy)$.

(Hint: use ∀E to get rid of the quantifiers, and then put them back on using ∀I.)