Exercises for 5.11-5.12

1. Add missing items (Hint: lines 6 and 11 are one of the more useful theorems of sentential logic):

1.	$\neg\forall x\neg Fx$	$\vdash \neg\forall x\neg Fx$	A
2.	$\neg\exists xFx$	$\vdash \neg\exists xFx$	A
3.	Fa	$\vdash Fa$	A
4.	Fa	$\vdash \exists xFx$	$\underline{3,\exists\text{I}}$
5.		$\vdash \underline{Fa \supset \exists xFx}$	4,⊃I
6.		$\vdash \underline{(Fa \supset \exists xFx) \supset (\neg\exists xFx \supset \neg Fa)}$	$\underline{\text{CP}}$
7.		$\vdash \neg\exists xFx \supset \neg Fa$	5,6,⊃E
8.	$\neg\exists xFx$	$\vdash \neg Fa$	$\underline{2,7,\supset\text{E}}$
9.	$\neg\exists xFx$	$\vdash \underline{\forall x\neg Fx}$	8,∀I
10.		$\vdash \neg\exists xFx \supset \forall x\neg Fx$	$\underline{9,\supset\text{I}}$
11.		$\vdash \underline{(\neg\exists xFx \supset \forall x\neg Fx) \supset (\neg\forall x\neg Fx \supset \neg\neg\exists xFx)}$	$\underline{\text{CP}}$
12.		$\vdash \neg\forall x\neg Fx \supset \neg\neg\exists xFx$	10,11, ⊃E
13.	$\neg\forall x\neg Fx$	$\vdash \neg\neg\exists xFx$	$\underline{1,\ 12,\ \supset\text{E}}$
14.	$\neg\forall x\neg Fx$	$\vdash \exists xFx$	13,¬E

2. Suppose it is true that $\forall x[(Fx \vee Gx) \supset Hx]$, and suppose that Fa, where a is a constant. It follows that Ha. Turn this into a derivation. Here are the first two lines:

1.	Γ	$\vdash \forall x[(Fx \vee Gx) \supset Hx]$	premise
2.	Δ	$\vdash Fa$	premise

3.	Δ	$\vdash Fa \vee Ga$	2,∨I
4.	Γ	$\vdash (Fa \vee Ga) \supset Ha$	1,∀E
5.	Γ, Δ	$\vdash Ha$	3,4,⊃E

3. Suppose everyone gets grumpy when hungry. So if everyone is hungry, everyone is grumpy. Turn this into a derivation. Here are the first two lines (Hx means x is hungry, Gx means x is grumpy; the conclusion you want to reach is $\Gamma \vdash \forall x Hx \supset \forall x Gx$):

1.	Γ	$\vdash \forall x(Hx \supset Gx)$	premise
2.	$\forall x Hx$	$\vdash \forall x Hx$	A

3.	$\forall x Hx$	$\vdash Ha$	2,∀E
4.	Γ	$\vdash Ha \supset Ga$	1,∀E
5.	$\Gamma, \forall x Hx$	$\vdash Ga$	3,4,⊃E
6.	$\Gamma, \forall x Hx$	$\vdash \forall x Gx$	5,∀I
7.	Γ	$\vdash \forall x Hx \supset \forall x Gx$	6,⊃I

4. Consider: No one is flawless; so everyone has flaws. Let's turn this into a derivation. Let Fx mean 'x has flaws' and let k be a constant. Hint: assume that k is not F.

1.	Γ	$\vdash \neg\exists x \neg Fx$	premise
2.	$\neg Fk$	$\vdash \neg Fk$	A
3.	$\neg Fk$	$\vdash \exists x \neg Fx$	2,∃I
4.	$\Gamma, \neg Fk$	$\vdash \neg\exists\neg Fx$	1
5.	Γ	$\vdash \neg\neg Fk$	3,4,¬I
6.	Γ	$\vdash Fk$	5,¬E
7.	Γ	$\vdash \forall x Fx$	6,∀I

5. Consider: dragons are mythical creatures; but there are no mythical creatures; thus, there are no dragons. Formalize this. Let Dx mean that x is a dragon, Mx mean that x is a mythical creature. Hint: derive something that enables you to use ¬I.

1.	Γ	$\vdash \forall x(Dx \supset Mx)$	premise
2.	Δ	$\vdash \neg\exists x Mx$	premise
3.	$\exists x Dx$	$\vdash \exists x Dx$	A
4.	Da	$\vdash Da$	A
5.	Γ	$\vdash Da \supset Ma$	1,∀E

6.	Γ, Da	$\vdash Ma$	4,5,⊃E
7.	Γ, Da	$\vdash \exists x Mx$	6,∃I
8.	$\Gamma, \exists x Dx$	$\vdash \exists x Mx$	3,7,∃E
9.	$\Delta, \exists x Dx$	$\vdash \neg\exists x Mx$	2
10.	Γ, Δ	$\vdash \neg\exists x Dx$	8,9,¬I

6. Prove $\vdash \forall x(Fx \supset \forall y Gy) \supset \forall x \forall y(Fx \supset Gy)$.

(Hint: use ∀E to get rid of the quantifiers, and then put them back on using ∀I.)

a and b are constants not appearing in Γ.

1.	$\forall x(Fx \supset \forall y Gy)$	$\vdash \forall x(Fx \supset \forall y Gy)$	A
2.	$\forall x(Fx \supset \forall y Gy)$	$\vdash Fa \supset \forall y Gy$	1,∀E
3.	Fa	$\vdash Fa$	A
4.	$\forall x(Fx \supset \forall y Gy), Fa$	$\vdash \forall y Gy$	2,3,⊃E
5.	$\forall x(Fx \supset \forall y Gy), Fa$	$\vdash Gb$	4,∀E
6.	$\forall x(Fx \supset \forall y Gy)$	$\vdash Fa \supset Gb$	5,⊃I
7.	$\forall x(Fx \supset \forall y Gy)$	$\vdash \forall y(Fa \supset Gy)$	6,∀I
8.	$\forall x(Fx \supset \forall y Gy)$	$\vdash \forall x \forall y(Fx \supset Gy)$	7,∀I