Exercises for 5.7 to 5.10

1. Consider: derivation: everyone gets grumpy when they are hungry; Erika is hungry; so someone is grumpy. Add the missing annotations in the formalization of this (e is a constant referring to Erika; Hx means 'x is hungry', Gx that 'x is grumpy'; e does not occur in Γ, Δ):

1.	Γ	$\vdash \forall x(Hx \supset Gx)$	..... premise
2.	Δ	$\vdash He$	..... premise
3.	Γ	$\vdash He \supset Ge$	..... 1,$\forall$E
4.	Γ, Δ	$\vdash Ge$	..... 2,3,$\supset$E
5.	Γ, Δ	$\vdash \exists x Gx$	..... 4,$\exists$I

2. Consider: All philosophy majors take logic. So if everyone majors in philosophy, everyone takes logic. Below is a formalization this. Add the missing annotations: (Px means 'x majors in philosophy', Lx means 'x takes logic'):

1.	Γ	$\vdash \forall x(Px \supset Lx)$	..... premise
2.	$\forall x Px$	$\vdash \forall x Px$	..... A
3.	$\forall x Px$	$\vdash Pa$	..... 2,$\forall$E
4.	Γ	$\vdash Pa \supset La$	..... 1,$\forall$E
5.	$\Gamma, \forall x Px$	$\vdash La$	..... 3,4,$\supset$E
6.	$\Gamma, \forall x Px$	$\vdash \forall x Lx$	..... 5,$\forall$I
7.	Γ	$\vdash \forall x Px \supset \forall x Lx$	..... 6,$\supset$I

3. Add missing annotations:

1.	Γ	$\vdash \exists x(Fx \wedge Gx)$	..... premise
2.	$Fa \wedge Ga$	$\vdash Fa \wedge Ga$	..... A
3.	$Fa \wedge Ga$	$\vdash Fa$	..... 2,$\wedge$E
4.	$Fa \wedge Ga$	$\vdash \exists x Fx$	..... 3,$\exists$I
5.	$Fa \wedge Ga$	$\vdash Ga$	..... 2,$\wedge$E
6.	$Fa \wedge Ga$	$\vdash \exists x Gx$	..... 5,$\exists$I
7.	$Fa \wedge Ga$	$\vdash \exists x Fx \wedge \exists x Gx$	..... 4,6,$\wedge$I

8. $\Gamma \vdash \exists x Fx \wedge \exists x Gx$ ..... 1,7,$\exists$E

4. What is wrong with the following attempted derivation?

1. $\Gamma \vdash \forall x(Fx \vee Gx)$ ..... premise
2. $\Gamma \vdash Fc \vee Gc$ ..... 1,$\forall$E
3. $Fc \vdash Fc$ ..... A
4. $Fc \vdash \forall x Fx$ ..... 3,$\forall$I
5. $Fc \vdash \forall x Fx \vee \forall x Gx$ ..... 4,$\vee$I
6. $Gc \vdash Gc$ ..... A
7. $Gc \vdash \forall x Gx$ ..... 6,$\forall$I
8. $Gc \vdash \forall x Fx \vee \forall x Gx$ ..... 7,$\vee$I
9. $\Gamma \vdash \forall x Fx \vee \forall x Gx$ ..... 2,5,8,$\vee$E

Answer Key

On lines 4 and 7, $\forall$I is misapplied as the rule requires that c not appear in any of the sentences on the datum side of lines 3 and 6 respectively.

5. Add missing annotations:

1. $\exists x(Fx \vee Gx) \vdash \exists x(Fx \vee Gx)$ ..... A
2. $Fa \vee Ga \vdash Fa \vee Ga$ ..... A
3. $Fa \vdash Fa$ ..... A
4. $Fa \vdash \exists x Fx$ ..... 3,$\exists$I
5. $Fa \vdash \exists x Fx \vee \exists x Gx$ ..... 4,$\vee$I
6. $Ga \vdash Ga$ ..... A
7. $Ga \vdash \exists x Gx$ ..... 6,$\exists$I
8. $Ga \vdash \exists x Fx \vee \exists x Gx$ ..... 7,$\vee$I
9. $Fa \vee Ga \vdash \exists x Fx \vee \exists x Gx$ ..... 2,5,8,$\vee$E
10. $\exists x(Fx \vee Gx) \vdash \exists x Fx \vee \exists x Gx$ ..... 1, 9, $\exists$E

6. Add missing items.

1.	$\forall x(Fx \supset Gx)$	$\vdash \forall x(Fx \supset Gx)$	A
2.	$\forall xFx$	$\vdash \forall xFx$	A
3.	$\forall xFx$	$\vdash Fa$	2,$\forall$E
4.	$\underline{\forall x(Fx \supset Gx)}$	$\vdash \underline{Fa \supset Ga}$	1,$\forall$E
5.	$\forall x(Fx \supset Gx), \forall xFx$	$\vdash Ga$	3,4,$\supset$E
6.	$\underline{\forall x(Fx \supset Gx), \forall xFx}$	$\vdash \underline{\forall xGx}$	5, $\forall$I
7.	$\underline{\forall x(Fx \supset Gx)}$	$\vdash \underline{\forall xFx \supset \forall xGx}$	6,$\supset$I
8.		$\vdash \forall x(Fx \supset Gx) \supset (\forall xFx \supset \forall xGx)$	7,$\supset$I

7. What's wrong with the following derivation (a is a constant not occurring in Γ):

1.	Γ	$\vdash \exists xFx \supset \exists xGx$	premise
2.	$\exists xFx$	$\vdash \exists xFx$	A
3.	$\Gamma, \exists xFx$	$\vdash \exists xGx$	1,2,$\supset$E
4.	Ga	$\vdash Ga$	A
5.	Fa, Ga	$\vdash Ga$	4
6.	Ga	$\vdash Fa \supset Ga$	5,$\supset$I
7.	Ga	$\vdash \exists x(Fx \supset Gx)$	6,$\exists$I
8.	Γ, Ga	$\vdash \exists x(Fx \supset Gx)$	7
9.	Γ	$\vdash \exists x(Fx \supset Gx)$	3,8,$\exists$E

Answer Key

The datum of the last line is wrong. It needs to be:

9.	$\Gamma, \exists xFx$	$\vdash \exists x(Fx \supset Gx)$	3,8,$\exists$E

8. Give an example illustrating why line 9 in the previous problem does not follow from line 1.

Here is one: If there are winners, then there are losers. It does not follow from that that there are some who when they win they lose.

9. Here is something you might be tempted to do: from $\Gamma \vdash \forall x(Fx \vee Gx)$ derive $\Gamma \vdash \forall xFx \vee \forall xGx$. Explain using an example why this would be a fallacious inference.

Let Fx mean that x is a Pomona student, and let Gx mean that x is a HMC student. It does not follow from that that everyone is a Pomona student or everyone is an HMC student.