Practicing Inference Rules

1. Each of the following little derivation has a mistake. What is it?

(a)
1. p $\vdash$ q ..... A

(b)
1. Γ $\vdash$ $p \vee q$ ..... premise
2. Γ $\vdash$ q ..... 1,$\vee$E

(c)
1. p $\vdash$ p ..... A
2. q $\vdash$ q ..... A
3. $p \wedge q$ $\vdash$ $p \wedge q$ ..... 1,2,$\wedge$I

(d)
1. Γ, s $\vdash$ r ..... premise
2. Γ, q $\vdash$ p ..... premise
3. Γ, s, q $\vdash$ $r \supset p$ ..... 1,2,$\wedge$I

(e)
1. s $\vdash$ s ..... A
2. $s \vee w$ $\vdash$ s ..... 1,$\vee$I

(f)
1. $p \wedge q$ $\vdash$ $p \wedge q$ ..... A
2. p $\vdash$ $p \wedge q$ ..... 1,$\wedge$E

(g)
1. r $\vdash$ $s \vee w$ ..... premise
2. r $\vdash$ $s \vee (w \vee q)$ ..... 1,$\vee$I

(h)
1. Γ, q $\vdash$ p ..... premise

2. Δ $\vdash \neg p$ . . . . . premise
3. Γ, Δ $\vdash \neg q$ . . . . . 1,2,¬I

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(i) 1. Γ $\vdash w$ . . . . . premise
2. Γ $\vdash \neg\neg w$ . . . . . 1,¬I

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(j) 1. Δ $\vdash \neg\neg p \vee r$ . . . . . premise
2. Δ $\vdash p \vee r$ . . . . . 1,¬E

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(k) 1. $p \supset q$ $\vdash p \supset q$ . . . . . A
2. q $\vdash q$ . . . . . A
3. $p \supset q, q$ $\vdash p$ . . . . . 1,2,⊃E

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(l) 1. $p \supset q$ $\vdash p \supset q$ . . . . . A
2. $p \supset q$ $\vdash p$ . . . . . 1,⊃E

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(m) 1. Γ $\vdash a$ . . . . . premise
2. Δ $\vdash b$ . . . . . premise
3. Γ, Δ $\vdash a \supset b$ . . . . . 1,2,⊃I

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2. Fill in missing items.

(i)
1. __ ⊢ __ ..... A
2. $p \supset q, r$ ⊢ $p \supset q$ 1

(ii)
1. __ ⊢ __ ..... A
2. $\Gamma, r \wedge s$ ⊢ $r \wedge s$ 1

(iii)
1. __ ⊢ __ ..... A
2. $\neg q \vee r, \neg\neg p$ ⊢ $\neg q \vee r$ 1

(iv)
1. Γ ⊢ $s \vee w$ ..... premise
2. __ ⊢ __ ..... premise
3. Γ, Δ ⊢ $(s \vee w) \wedge (p \supset q)$ ..... 1,2,∧I

(v)
1. Γ ⊢ p ..... premise
2. __ ⊢ __ ..... 1,∨I
3. __ ⊢ __ ..... premise
4. Γ, Δ ⊢ $(p \vee q) \wedge \neg s$ ..... 2,3,∧I

(vi)
1. Γ ⊢ $q \wedge r$ ..... premise
2. __ ⊢ __ ..... 1,∧E
3. Γ ⊢ $s \vee q$ ..... 2,∨I

(vii)
1. Γ ⊢ s ..... premise
2. Δ ⊢ r ..... premise
3. Γ ⊢ __ ..... 1,∨I
4. Γ, Δ ⊢ $(s \vee w) \wedge r$ ..... 2,3,∧I

(viii)

1.	Γ	$\vdash s$	..... premise
2.	Δ	$\vdash r$	..... premise
3.	__	$\vdash$ __	..... 1,$\vee$I
4.	__	$\vdash$ __	..... 2,$\vee$I
5.	Γ, Δ	$\vdash (s \vee w) \wedge (\neg p \vee r)$	..... 3,4,$\wedge$I

(ix)

1.	Γ	$\vdash \neg p$	..... premise
2.	Γ, s	$\vdash \neg p$	 1
3.	__	$\vdash$ __	..... 2,$\supset$I

(x)

1.	Γ	$\vdash u \wedge w$	..... premise
2.	__	$\vdash$ __	..... 1,$\wedge$E
3.	__	$\vdash$ __	 2
4.	Γ	$\vdash s \supset u$	..... 3,$\supset$I

(xi)

1.	Γ	$\vdash a \supset b$	..... premise
2.	Δ	$\vdash a$	..... premise
3.	__	$\vdash$ __	..... 1,2,$\supset$E

(xii)

1.	Γ	$\vdash$ __	..... premise
2.	__	$\vdash x$	..... premise
3.	Γ, Δ	$\vdash y$	..... 1,2,$\supset$E

(xiii)

1.	Γ	$\vdash \neg\neg p$	..... premise
2.	__	$\vdash$ __	..... 1,$\neg$E

(xiv)

1.	Δ	$\vdash$ __	..... premise
2.	__	$\vdash s$	..... 1,$\neg$E

(xv)
1. $\Gamma, q \vdash \neg(p \vee s)$ ..... premise
2. $\Delta, q \vdash p \vee s$ ..... premise
3. $\_\_ \vdash \_\_$ ..... 1,2,¬I

(xvi)
1. $\Gamma, p \vdash q \supset r$ ..... premise
2. $\Delta, p \vdash \_\_$ ..... premise
3. $\Gamma, \Delta \vdash \neg p$ ..... 1,2,¬I

(xvii)
1. $\Gamma, \_\_ \vdash q \vee \neg r$ ..... premise
2. $\Delta, \_\_ \vdash \_\_$ ..... premise
3. $\Gamma, \Delta \vdash \neg(s \wedge w)$ ..... 1,2,¬I

(xviii)
1. $\Gamma \vdash q \vee r$ ..... premise
2. $\Delta, q \vdash p \supset s$ ..... premise
3. $\Theta, r \vdash p \supset s$ ..... premise
4. $\_\_ \vdash \_\_$ ..... 1,2,3,∨E

(xix)
1. $\Gamma, p \vdash w \wedge (p \supset q)$ ..... premise
2. $\Delta \vdash p \vee q$ ..... premise
3. $\Theta, q \vdash w \wedge (p \supset q)$ ..... premise
4. $\_\_ \vdash \_\_$ ..... 1,2, 3,∨E

(xx)
1. $\Gamma \vdash \_\_$ ..... premise
2. $\Delta, x \vdash w$ ..... premise
3. $\Theta, y \vdash w$ ..... premise
4. $\Gamma, \Delta, \Theta \vdash w$ ..... 1,2,3,∨E

(xxi)
1. $\Gamma \vdash r \vee s$ ..... premise
2. $\Delta, \_\_ \vdash \_\_$ ..... premise
3. $\Theta, s \vdash \_\_$ ..... premise
4. $\Gamma, \Delta, \Theta \vdash \neg(p \wedge q)$ ..... 1,2,3,∨E

3. The following have blanks that are impossible to fill in in accordance with our inference rules if we stick to the annotations. Explain why.

(a)
1. __ $\vdash r$ . . . . . A
2. p, q $\vdash p$ 1

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(b)
1. r $\vdash r$ . . . . . A
2. __ $\vdash$ __ 1
3. r $\vdash p \supset q$ . . . . . 2,⊃I

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(c)
1. Γ $\vdash r$ . . . . . premise
2. s $\vdash s$ . . . . . A
3. s $\vdash$ __ . . . . . 2,∨I
4. Γ, s $\vdash r \wedge (s \wedge q)$ . . . . . 1,3,∧I

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(d)
1. Γ $\vdash \neg\neg p$ . . . . . premise
2. __ $\vdash$ __ . . . . . 1,¬E
3. Δ $\vdash p \supset q$ . . . . . premise
4. Δ $\vdash q$ . . . . . 2,3,⊃E

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(e)

1.	Θ	$\vdash$ __	premise
2.	Γ, p	$\vdash r$	premise
3.	Δ, q	$\vdash r$	premise
4.	Γ, Δ, Θ	$\vdash r$	1,2,3,$\vee$E
5.	Θ	$\vdash$ __	1,$\wedge$E

(f)

1.	Γ	$\vdash$ __	premise
2.	Δ	$\vdash$ __	premise
3.	Θ, p	$\vdash q$	premise
4.	Γ, Δ, Θ	$\vdash$ __	1,2,$\wedge$I
5.	$\Gamma, \Delta, \Theta, p$	$\vdash$ __	4
6.	Γ, Δ, Θ	$\vdash \neg p$	3,5,$\neg$I

(g)

1.	Γ	$\vdash \neg\neg p$	premise
2.	Γ	$\vdash$ __	1,$\vee$I
3.	Γ	$\vdash p \vee q$	2,$\neg$E