Practicing Inference Rules

1. Each of the following little derivation has a mistake. What is it?

(a) 1. $p \vdash q$ . . . . . A

> The datum and succedent cannot differ when using A

(b) 1. $\Gamma \vdash p \vee q$ . . . . . premise
2. $\Gamma \vdash q$ . . . . . 1,$\vee$E

> $\vee$E cannot be used in that way.

(c) 1. $p \vdash p$ . . . . . A
2. $q \vdash q$ . . . . . A
3. $p \wedge q \vdash p \wedge q$ . . . . . 1,2,$\wedge$I

> $\wedge$I does not result in that datum.

(d) 1. $\Gamma, s \vdash r$ . . . . . premise
2. $\Gamma, q \vdash p$ . . . . . premise
3. $\Gamma, s, q \vdash r \supset p$ . . . . . 1,2,$\wedge$I

> $\wedge$I results in a succedent with $\wedge$ as main connective.

(e) 1. $s \vdash s$ . . . . . A
2. $s \vee w \vdash s$ . . . . . 1,$\vee$I

> $\vee$I does not change datum.

(f) 1. $p \wedge q \vdash p \wedge q$ . . . . . A
2. $p \vdash p \wedge q$ . . . . . 1,$\wedge$E

> Cannot eliminate a conjunct from a conjunction in datum using $\wedge$E.

(g) 1. $r \vdash s \vee w$ . . . . . premise
2. $r \vdash s \vee (w \vee q)$ . . . . . 1,$\vee$I

Cannot introduce a disjunct in that way using ∨I.

(h)

1. $\Gamma, q \vdash p$ . . . . . premise
2. $\Delta \vdash \neg p$ . . . . . premise
3. $\Gamma, \Delta \vdash \neg q$ . . . . . 1,2,¬I

q is not in the datum of line 2.

(i)

1. $\Gamma \vdash w$ . . . . . premise
2. $\Gamma \vdash \neg\neg w$ . . . . . 1,¬I

Cannot introduce double negation using ¬I.

(j)

1. $\Delta \vdash \neg\neg p \vee r$ . . . . . premise
2. $\Delta \vdash p \vee r$ . . . . . 1,¬E

Can't eliminate the double negation in this way.

(k)

1. $p \supset q \vdash p \supset q$ . . . . . A
2. $q \vdash q$ . . . . . A
3. $p \supset q, q \vdash p$ . . . . . 1,2,⊃E

Not how ⊃E works.

(l)

1. $p \supset q \vdash p \supset q$ . . . . . A
2. $p \supset q \vdash p$ . . . . . 1,⊃E

Not how ⊃E works

(m)

1. $\Gamma \vdash a$ . . . . . premise
2. $\Delta \vdash b$ . . . . . premise
3. $\Gamma, \Delta \vdash a \supset b$ . . . . . 1,2,⊃I

Not how ⊃I works.

2. Fill in missing items.

(i)
1. $\underline{p \supset q}$ $\vdash$ $\underline{p \supset q}$ ..... A
2. $p \supset q, r$ $\vdash$ $p \supset q$ 1

(ii)
1. $\underline{r \wedge s}$ $\vdash$ $\underline{r \wedge s}$ ..... A
2. $\Gamma, r \wedge s$ $\vdash$ $r \wedge s$ 1

(iii)
1. $\underline{\neg q \vee r}$ $\vdash$ $\underline{\neg q \vee r}$ ..... A
2. $\neg q \vee r, \neg\neg p$ $\vdash$ $\neg q \vee r$ 1

(iv)
1. Γ $\vdash$ $s \vee w$ ..... premise
2. $\underline{\Delta}$ $\vdash$ $\underline{p \supset q}$ ..... premise
3. Γ, Δ $\vdash$ $(s \vee w) \wedge (p \supset q)$ ..... 1,2,$\wedge$I

(v)
1. Γ $\vdash$ p ..... premise
2. $\underline{\Gamma}$ $\vdash$ $\underline{p \vee q}$ ..... 1,$\vee$I
3. $\underline{\Delta}$ $\vdash$ $\underline{\neg s}$ ..... premise
4. Γ, Δ $\vdash$ $(p \vee q) \wedge \neg s$ ..... 2,3,$\wedge$I

(vi)
1. Γ $\vdash$ $q \wedge r$ ..... premise
2. $\underline{\Gamma}$ $\vdash$ $\underline{q}$ ..... 1,$\wedge$E
3. Γ $\vdash$ $s \vee q$ ..... 2,$\vee$I

(vii)
1. Γ $\vdash$ s ..... premise
2. Δ $\vdash$ r ..... premise
3. Γ $\vdash$ $\underline{s \vee w}$ ..... 1,$\vee$I
4. Γ, Δ $\vdash$ $(s \vee w) \wedge r$ ..... 2,3,$\wedge$I

(viii)

1.	Γ	$\vdash s$	premise
2.	Δ	$\vdash r$	premise
3.	$\underline{\Gamma}$	$\vdash \underline{s \vee w}$	1,$\vee$I
4.	$\underline{\Delta}$	$\vdash \underline{\neg p \vee r}$	2,$\vee$I
5.	Γ, Δ	$\vdash (s \vee w) \wedge (\neg p \vee r)$	3,4,$\wedge$I

(ix)

1.	Γ	$\vdash \neg p$	premise
2.	Γ, s	$\vdash \neg p$	1
3.	$\underline{\Gamma}$	$\vdash \underline{s \supset \neg p}$	2,$\supset$I

(x)

1.	Γ	$\vdash u \wedge w$	premise
2.	$\underline{\Gamma}$	$\vdash \underline{u}$	1,$\wedge$E
3.	$\underline{\Gamma, s}$	$\vdash \underline{u}$	2
4.	Γ	$\vdash s \supset u$	3,$\supset$I

(xi)

1.	Γ	$\vdash a \supset b$	premise
2.	Δ	$\vdash a$	premise
3.	$\underline{\Gamma, \Delta}$	$\vdash \underline{b}$	1,2,$\supset$E

(xii)

1.	Γ	$\vdash \underline{x \supset y}$	premise
2.	$\underline{\Delta}$	$\vdash x$	premise
3.	Γ, Δ	$\vdash y$	1,2,$\supset$E

(xiii)

1.	Γ	$\vdash \neg\neg p$	premise
2.	$\underline{\Gamma}$	$\vdash \underline{p}$	1,$\neg$E

(xiv)

1.	Δ	$\vdash \underline{\neg\neg s}$	premise
2.	$\underline{\Delta}$	$\vdash s$	1,$\neg$E

(xv)
1. $\Gamma, q \vdash \neg(p \vee s)$ ..... premise
2. $\Delta, q \vdash p \vee s$ ..... premise
3. $\underline{\Gamma, \Delta} \vdash \underline{\neg q}$ ..... 1,2,¬I

(xvi)
1. $\Gamma, p \vdash q \supset r$ ..... premise
2. $\Delta, p \vdash \underline{\neg(q \supset r)}$ ..... premise
3. $\Gamma, \Delta \vdash \neg p$ ..... 1,2,¬I

(xvii)
1. $\Gamma, \underline{s \wedge w} \vdash q \vee \neg r$ ..... premise
2. $\Delta, \underline{s \wedge w} \vdash \underline{\neg(q \vee \neg r)}$ ..... premise
3. $\Gamma, \Delta \vdash \neg(s \wedge w)$ ..... 1,2,¬I

(xviii)
1. $\Gamma \vdash q \vee r$ ..... premise
2. $\Delta, q \vdash p \supset s$ ..... premise
3. $\Theta, r \vdash p \supset s$ ..... premise
4. $\underline{\Gamma, \Delta, \Theta} \vdash \underline{p \supset s}$ ..... 1,2,3,∨E

(xix)
1. $\Gamma, p \vdash w \wedge (p \supset q)$ ..... premise
2. $\Delta \vdash p \vee q$ ..... premise
3. $\Theta, q \vdash w \wedge (p \supset q)$ ..... premise
4. $\underline{\Gamma, \Delta, \Theta} \vdash \underline{w \wedge (p \supset q)}$ ..... 1,2, 3,∨E

(xx)
1. $\Gamma \vdash \underline{x \vee y}$ ..... premise
2. $\Delta, x \vdash w$ ..... premise
3. $\Theta, y \vdash w$ ..... premise
4. $\Gamma, \Delta, \Theta \vdash w$ ..... 1,2,3,∨E

(xxi)
1. $\Gamma \vdash r \vee s$ ..... premise
2. $\Delta, \underline{r} \vdash \underline{\neg(p \wedge q)}$ ..... premise
3. $\Theta, s \vdash \underline{\neg(p \wedge q)}$ ..... premise
4. $\Gamma, \Delta, \Theta \vdash \neg(p \wedge q)$ ..... 1,2,3,∨E

3. The following have blanks that are impossible to fill in in accordance with our inference rules if we stick to the annotations. Explain why.

(a)
1. __ $\vdash r$ . . . . . A
2. p, q $\vdash p$ 1

> The blank would have to be an r given the annotation of line 1, but you can't get line 2 from that in the way indicated.

(b)
1. r $\vdash r$ . . . . . A
2. __ $\vdash$ __ 1
3. r $\vdash p \supset q$ . . . . . 2,$\supset$I

> Given the annotation of line 3, line 2 must have a q in succedent, but you can't get that given line 2 is said to be a rewrite of line 1.

(c)
1. Γ $\vdash r$ . . . . . premise
2. s $\vdash s$ . . . . . A
3. s $\vdash$ __ . . . . . 2,$\vee$I
4. Γ, s $\vdash r \wedge (s \wedge q)$ . . . . . 1,3,$\wedge$I

> Given the annotation of line 4, the succedent of line 3 must be $s \wedge q$, but you can't get that since the annotation of line 3 requires that the succedent be a disjunction.

(d)
1. Γ $\vdash \neg\neg p$ . . . . . premise
2. __ $\vdash$ __ . . . . . 1,$\neg$E
3. Δ $\vdash p \supset q$ . . . . . premise
4. Δ $\vdash q$ . . . . . 2,3,$\supset$E

> Given the annotation of line 4, the datum of line 2 must either be empty or Δ. But the annotation of 2 requires that the datum be Γ.

(e)

1.	Θ	$\vdash$ __	..... premise
2.	Γ, p	$\vdash r$	.....premise
3.	Δ, q	$\vdash r$	.....premise
4.	Γ, Δ, Θ	$\vdash r$	.....1,2,3,$\vee$E
5.	Θ	$\vdash$ __	.....1,$\wedge$E

According to the annoation of line 5, line 1's succedent must be a conjunction, but the annotation of line 4 requires it to be a conjunction.

(f)

1.	Γ	$\vdash$ __	..... premise
2.	Δ	$\vdash$ __	..... premise
3.	Θ, p	$\vdash q$	.....premise
4.	Γ, Δ, Θ	$\vdash$ __	.....1,2,$\wedge$I
5.	$\Gamma, \Delta, \Theta, p$	$\vdash$ __	4
6.	Γ, Δ, Θ	$\vdash \neg p$	..... 3,5,$\neg$I

Given the annotation of line 6, the succedents of lines 4 and 5 must be $\neg q$. But given the annotation of line 4, the succedent must be a conjunction.

(g)

1.	Γ	$\vdash \neg\neg p$	.....premise
2.	Γ	$\vdash$ __	..... 1,$\vee$I
3.	Γ	$\vdash p \vee q$	..... 2,$\neg$E

Given the annotation of line 2, the main connective of its succedent must be $\vee$, but then you cannot use $\neg$E to infer line 3 from line 2.