Inference Rules for Predicate Logic

Fill in blanks.

1. 1. $\neg\exists x Fx \vdash \neg Fk$ ..... premise
   2. $\neg\exists x Fx \vdash \underline{\forall x \neg Fx}$ ..... 1,∀I

2. 1. $Fk \supset Gk \vdash Hc$ ..... premise
   2. $\underline{Fk \supset Gk} \vdash \forall x Hx$ ..... 1,∀I

3. 1. $\exists x(Gx \vee Hx) \vdash Fa$ ..... premise
   2. $\underline{\exists x(Gx \vee Hx)} \vdash \underline{\forall x Fx}$ ..... 1,∀I

4. 1. $Fa \vdash \forall x(Hx \supset Px)$ ..... premise
   2. $Fa \vdash \underline{Ha \supset Pa}$ ..... 1,∀E
   3. $Ha \vdash Ha$ ..... A
   4. $Fa, Ha \vdash Pa$ ..... 2,3,⊃E

5. 1. $Fa \vdash \forall x(Hx \supset Px)$ ..... premise
   2. $\underline{Fa} \vdash \underline{Hb \supset Pb}$ ..... 1,∀E
   3. $Hb \vdash Hb$ ..... A
   4. $Fa, Hb \vdash Pb$ ..... 2,3,⊃E

6. 1. $\exists x Qx \vdash \exists x Qx$ ..... A
   2. $Qa \vdash \exists x Rx$ ..... premise
   3. $\underline{\exists x Qx} \vdash \underline{\exists x Rx}$ ..... 1,2,∃E

7. 1. $Qb \vdash Qb$ ..... A
   2. $\underline{Qb} \vdash \underline{\exists x Qx}$ ..... 1,∃I

8.

1.	Γ	$\vdash \exists z W z \supset \exists z M z$	..... premise
2.	Δ	$\vdash W a$	..... premise
3.	Δ	$\vdash \underline{\exists z W z}$	.....2,∃I
4.	Γ, Δ	$\vdash \exists z M z$	.....1,3,⊃E

9.

1.	$\exists x F x$	$\vdash \exists x F x$	..... premise
2.	$F a$	$\vdash F a$	.....A
3.	$\underline{F a}$	$\vdash \underline{\exists y F y}$	..... 2,∃I
4.	$\exists x F x$	$\vdash \exists y F y$	.....1,3,∃E

10.

1.	$\exists x(Rx \vee Fx)$	$\vdash \exists x(Rx \vee Fx)$	..... A
2.	$Ra \vee Fa$	$\vdash Ra \vee Fa$	..... A
3.	$\forall x[(Rx \vee Fx) \supset Wx]$	$\vdash \forall x[(Rx \vee Fx) \supset Wx]$	A
4.	$\forall x[(Rx \vee Fx) \supset Wx]$	$\vdash \underline{(Ra \vee Fa) \supset Wa}$	.. 3,∀E
5.	$\forall x[(Rx \vee Fx) \supset Wx], Ra \vee Fa$	$\vdash Wa$	.....2,4,⊃E
6.	$\forall x[(Rx \vee Fx) \supset Wx], Ra \vee Fa$	$\vdash \exists x Wx$	..... 5,∃I
7.	$\underline{\forall x[(Rx \vee Fx) \supset Wx], \exists x(Rx \vee Fx)}$	$\vdash \underline{\exists x Wx}$	..... 1,6,∃E

11.

1.	$\exists x \forall y L y x$	$\vdash \exists x \forall y L y x$	..... A
2.	$\forall y L y r$	$\vdash \forall y L y r$	..... A
3.	$\forall y L y r$	$\vdash L d r$	..... 2,∀E
4.	$\forall y L y r$	$\vdash \exists x L d x$	..... 3,∃I
5.	$\forall y L y r$	$\vdash \underline{\forall y \exists x L y x}$	..... 4,∀I
6.	$\underline{\exists x \forall y L y x}$	$\vdash \forall y \exists x L y x$	.....1, 5,∃E