Note: A derivation in predicate logic need not use any of the quantifier-specific rules. The first four problems below are examples of this point.

1. Add missing annotations:

| | | | |
|---|---|---|---|
| 1. | $\Gamma$ | $\vdash Fc \vee Gd$ | ..... premise |
| 2. | $\Delta$ | $\vdash Fc \supset Fk$ | ..... premise |
| 3. | $\Theta$ | $\vdash Gd \supset He$ | ..... premise |
| 4. | $Fc$ | $\vdash Fc$ | ..... A |
| 5. | $\Delta, Fc$ | $\vdash Fk$ | ..... 2,4,⊃E |
| 6. | $\Delta, Fc$ | $\vdash Fk \vee He$ | ..... 5,∨I |
| 7. | $Gd$ | $\vdash Gd$ | ..... A |
| 8. | $\Theta, Gd$ | $\vdash He$ | ..... 3,7,⊃E |
| 9. | $\Theta, Gd$ | $\vdash Fk \vee He$ | ..... 8,∨I |
| 10. | $\Gamma, \Delta, \Theta$ | $\vdash Fk \vee He$ | ..... 1,6,9,∨E |

2. Add missing annotations:

| | | | |
|---|---|---|---|
| 1. | $\Gamma$ | $\vdash \forall xFx \vee \exists xGx$ | ..... premise |
| 2. | $\Delta$ | $\vdash \forall xFx \supset \neg\forall xFx$ | ..... premise |
| 3. | $\Theta$ | $\vdash \exists xGx \supset \forall xHx$ | ..... premise |
| 4. | $\forall xFx$ | $\vdash \forall xFx$ | ..... A |
| 5. | $\Delta, \forall xFx$ | $\vdash \neg\forall xFx$ | ..... 2,4,⊃E |
| 6. | $\Delta, \forall xFx$ | $\vdash \neg\forall xFx \vee \forall xHx$ | ..... 5,∨I |
| 7. | $\exists xGx$ | $\vdash \exists xGx$ | ..... A |
| 8. | $\Theta, \exists xGx$ | $\vdash \forall xHx$ | ..... 3,7,⊃E |
| 9. | $\Theta, \exists xGx$ | $\vdash \neg\forall xFx \vee \forall xHx$ | ..... 8,∨I |
| 10. | $\Gamma, \Delta, \Theta$ | $\vdash \neg\forall xFx \vee \forall xHx$ | ..... 1,6,9,∨E |

3. Fill in the missing items.

| | | | |
|---|---|---|---|
| 1. | $\Gamma$ | $\vdash Fa \vee (Lb \vee Kc)$ | ..... premise |
| 2. | $Fa$ | $\vdash Fa$ | ..... A |
| 3. | $\underline{Kc}$ | $\vdash \underline{Kc}$ | ..... A |
| 4. | $Kc$ | $\vdash (Fa \vee Lb) \vee Kc$ | ..... 3,$\underline{\vee \text{I}}$ |
| 5. | $\underline{Fa}$ | $\vdash \underline{Fa \vee Lb}$ | ..... 2,$\vee$I |
| 6. | $\underline{Lb}$ | $\vdash \underline{Lb}$ | ..... A |
| 7. | $Lb$ | $\vdash \underline{Fa \vee Lb}$ | ..... 6,$\underline{\vee \text{I}}$ |
| 8. | $\underline{Fa}$ | $\vdash \underline{(Fa \vee Lb) \vee Kc}$ | ..... 5,$\vee$I |
| 9. | $Lb \vee Kc$ | $\vdash Lb \vee Kc$ | ..... A |
| 10. | $\underline{Lb}$ | $\vdash (Fa \vee Lb) \vee Kc$ | ..... 7,$\vee$I |
| 11. | $\underline{Lb \vee Kc}$ | $\vdash \underline{(Fa \vee Lb) \vee Kc}$ | ..... $\underline{4,9,10,\vee \text{E}}$ |
| 12. | $\Gamma$ | $\vdash (Fa \vee Lb) \vee Kc$ | ..... 1,8,11,$\vee$E |

4. Fill in the missing items.

| | | | |
|---|---|---|---|
| 1. | $\Gamma$ | $\vdash \exists x Fx \vee (\forall x Lx \vee \exists x Kx)$ | ..... premise |
| 2. | $\exists x Fx$ | $\vdash \exists x Fx$ | ..... A |
| 3. | $\underline{\exists x Kx}$ | $\vdash \underline{\exists x Kx}$ | ..... A |
| 4. | $\exists x Kx$ | $\vdash (\exists x Fx \vee \forall x Lx) \vee \exists x Kx$ | ..... 3,$\underline{\vee \text{I}}$ |
| 5. | $\underline{\exists x Fx}$ | $\vdash \underline{\exists x Fx \vee \forall x Lx}$ | ..... 2,$\vee$I |
| 6. | $\underline{\forall x Lx}$ | $\vdash \underline{\forall x Lx}$ | ..... A |
| 7. | $\forall x Lx$ | $\vdash \underline{\exists x Fx \vee \forall x Lx}$ | ..... 6,$\underline{\vee \text{I}}$ |
| 8. | $\underline{\exists x Fx}$ | $\vdash \underline{(\exists x Fx \vee \forall x Lx) \vee \exists x Kx}$ | ..... 5,$\vee$I |
| 9. | $\forall x Lx \vee \exists x Kx$ | $\vdash \forall x Lx \vee \exists x Kx$ | ..... A |
| 10. | $\underline{\forall x Lx}$ | $\vdash (\exists x Fx \vee \forall x Lx) \vee \exists x Kx$ | ..... 7,$\vee$I |
| 11. | $\underline{\forall x Lx \vee \exists x Kx}$ | $\vdash \underline{(\exists x Fx \vee \forall x Lx) \vee \exists x Kx}$ | ..... $\underline{4,9,10,\vee \text{E}}$ |
| 12. | $\Gamma$ | $\vdash (\exists x Fx \vee \forall x Lx) \vee \exists x Kx$ | ..... 1,8,11,$\vee$E |

5. Fill in missing items.

1. $\underline{\forall x Fx} \vdash \forall x Fx$ ..... A
2. $\forall x Gx \vdash \underline{\forall x Gx}$ ..... A
3. $\underline{\forall x Fx} \vdash \underline{Fa}$ ..... 1,$\forall$E
4. $\underline{\forall x Gx} \vdash \underline{Ga}$ ..... 2,$\forall$E
5. $\underline{\forall x Fx, \forall x Gx} \vdash Fa \wedge Ga$ ..... 3,4, $\wedge$I
6. $\forall x Fx, \forall x Gx \vdash \forall x(Fx \wedge Gx)$ ..... $\underline{5,\forall I}$

6. Fill in missing items.

1. $\underline{\forall x Fx \vee \forall x Gx} \vdash \underline{\forall x Fx \vee \forall x Gx}$ ..... A
2. $\underline{\forall x Fx} \vdash \forall x Fx$ ..... A
3. $\underline{\forall x Fx} \vdash \underline{Fk}$ ..... 2,$\forall$E
4. $\underline{\forall x Fx} \vdash \underline{Fk} \vee Gk$ ..... 3,$\vee$I
5. $\forall x Gx \vdash \forall x Gx$ ..... $\underline{A}$
6. $\underline{\forall x Gx} \vdash \underline{Gk}$ ..... 5,$\forall$E
7. $\underline{\forall x Gx} \vdash Fk \vee \underline{Gk}$ ..... 6,$\vee$I
8. $\forall x Fx \vee \forall x Gx \vdash Fk \vee Gk$ ..... 1,4,7,$\vee$E
9. $\underline{\forall x Fx \vee \forall x Gx} \vdash \forall x(Fx \vee Gx)$ ..... 8,$\forall$I

7. Fill in missing items.

1. $\forall x Fx \vdash \forall x Fx$ ..... $\underline{A}$
2. $\forall x Fx \vdash Fa$ ..... $\underline{1,\forall E}$
3. $\forall x Fx \vdash \exists x Fx$ ..... $\underline{2,\exists I}$

8. Fill in missing items.

| | | | |
|---|---|---|---|
| 1. | $\neg\forall xFx$ | $\vdash \neg\forall xFx$ | A |
| 2. | $\neg\exists x\neg Fx$ | $\vdash \neg\exists x\neg Fx$ | A |
| 3. | $\underline{\neg Fa}$ | $\vdash \underline{\neg Fa}$ | A |
| 4. | $\underline{\neg Fa}$ | $\vdash \underline{\exists x\neg Fx}$ | 3,∃I |
| 5. | $\underline{\neg\exists x\neg Fx, \neg Fa}$ | $\vdash \underline{\neg\exists x\neg Fx}$ | 2 |
| 6. | $\neg\exists x\neg Fx$ | $\vdash \underline{\neg\neg Fa}$ | 4,5,¬I |
| 7. | $\neg\exists x\neg Fx$ | $\vdash Fa$ | 6,¬E |
| 8. | $\neg\exists x\neg Fx$ | $\vdash \forall xFx$ | $\underline{7,\forall I}$ |
| 9. | $\neg\forall xFx, \neg\exists x\neg Fx$ | $\vdash \neg\forall xFx$ | $\underline{1}$ |
| 10. | $\neg\forall xFx$ | $\vdash \neg\neg\exists x\neg Fx$ | $\underline{8,\ 9,\neg I}$ |
| 11. | $\neg\forall xFx$ | $\vdash \exists x\neg Fx$ | 10,¬E |

9. Fill in missing items.

| | | | |
|---|---|---|---|
| 1. | $\neg\exists xFx$ | $\vdash \neg\exists xFx$ | A |
| 2. | $\underline{Fa}$ | $\vdash \underline{Fa}$ | A |
| 3. | $\underline{Fa}$ | $\vdash \underline{\exists xFx}$ | 2,∃I |
| 4. | $\underline{\neg\exists xFx, Fa}$ | $\vdash \underline{\neg\exists xFx}$ | 1 |
| 5. | $\neg\exists xFx$ | $\vdash \neg Fa$ | 3,4,¬I |
| 6. | $\neg\exists xFx$ | $\vdash \exists x\neg Fx$ | 5,∃I |

10. Fill in missing items. Hint: Line 1 is a theorem of sentential logic. Lines 2 and 3 are simply citing the results of Q3 and Q4; leave them untouched.

| | | | |
|---|---|---|---|
| 1. | | $\vdash \underline{\forall xFx \vee \neg\forall xFx}$ | $\underline{\text{EM}}$ |
| 2. | $\forall xFx$ | $\vdash \exists xFx$ | Q3 above |
| 3. | $\neg\forall xFx$ | $\vdash \exists x\neg Fx$ | Q4 above |
| 4. | $\forall xFx$ | $\vdash \underline{\exists xFx \vee \exists x\neg Fx}$ | $\underline{2,\vee I}$ |
| 5. | $\neg\forall xFx$ | $\vdash \underline{\exists xFx \vee \exists x\neg Fx}$ | $\underline{3,\vee I}$ |
| 6. | | $\vdash \exists xFx \vee \exists x\neg Fx$ | 1,4,5,∨E |

11. Fill in missing items.

1. $\forall x(Fx \supset Gx) \vdash \forall x(Fx \supset Gx)$ ..... A
2. $\underline{\forall x \neg Gx} \vdash \underline{\forall x \neg Gx}$ ..... A
3. $\underline{\forall x(Fx \supset Gx)} \vdash \underline{Fa \supset Ga}$ ..... 1, $\forall$E
4. $\underline{Fa} \vdash \underline{Fa}$ ..... A
5. $\forall x(Fx \supset Gx), Fa \vdash Ga$ ..... 3,4,$\supset$E
6. $\underline{\forall x \neg Gx} \vdash \neg Ga$ ..... 2,$\forall$E
7. $\underline{\forall x \neg Gx, Fa} \vdash \neg Ga$ ..... 6
8. $\forall x(Fx \supset Gx), \forall x \neg Gx \vdash \neg Fa$ ..... 5,7,$\neg$I
9. $\underline{\forall x(Fx \supset Gx), \forall x \neg Gx} \vdash \forall x \neg Fx$ ..... 8,$\forall$I

12. Fill in missing items.

1. $\exists xFx \vee \exists xGx \vdash \exists xFx \vee \exists xGx$ ..... A
2. $\underline{\exists xFx} \vdash \underline{\exists xFx}$ ..... A
3. $\underline{Fa} \vdash \underline{Fa}$ ..... A
4. $\underline{Fa} \vdash \underline{Fa \vee Ga}$ ..... 3,$\vee$I
5. $Fa \vdash \underline{\exists x(Fx \vee Gx)}$ ..... 4,$\exists$I
6. $\exists xFx \vdash \exists x(Fx \vee Gx)$ ..... 2,5,$\exists$E
7. $\exists xGx \vdash \exists xGx$ ..... A
8. $\underline{Gb} \vdash \underline{Gb}$ ..... A
9. $\underline{Gb} \vdash \underline{Fb \vee Gb}$ ..... 8,$\vee$I
10. $\underline{Gb} \vdash \underline{\exists x(Fx \vee Gx)}$ ..... 9,$\exists$I
11. $\exists xGx \vdash \exists x(Fx \vee Gx)$ ..... 7,10,$\exists$E
12. $\exists xFx \vee \exists xGx \vdash \exists x(Fx \vee Gx)$ ..... $\underline{1,6,11,\vee E}$

13.

| | | | |
|---|---|---|---|
| 1. | $\exists x Fx \vee \exists x Gx$ | $\vdash \exists x Fx \vee \exists x Gx$ | A |
| 2. | $\forall x(Fx \supset Gx)$ | $\vdash \forall x(Fx \supset Gx)$ | A |
| 3. | $\exists x Fx$ | $\vdash \exists x Fx$ | A |
| 4. | $Fa$ | $\vdash Fa$ | A |
| 5. | $\forall x(Fx \supset Gx)$ | $\vdash Fa \supset Ga$ | 2, $\forall$E |
| 6. | $\forall x(Fx \supset Gx), Fa$ | $\vdash Ga$ | 4,5,$\supset$E |
| 7. | $\forall x(Fx \supset Gx), Fa$ | $\vdash \exists x Gx$ | 6, $\exists$I |
| 8. | $\forall x(Fx \supset Gx), \exists x Fx$ | $\vdash \exists x Gx$ | 3,7,$\exists$E |
| 9. | $\exists x Gx$ | $\vdash \exists x Gx$ | A |
| 10. | $\exists x Fx \vee \exists x Gx, \forall x(Fx \supset Gx)$ | $\vdash \exists x Gx$ | 1,8,9,$\vee$E |